Uncertainties for S-parameter measurements in coaxial line and waveguide

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Outline

- What do we mean by “accuracy”?
- Why is “accuracy” important?
- How to determine “accuracy”
- Practical example
  - S-parameters (in coax or waveguide) using a VNA
- Conclusions
What do we mean by “accuracy”?

- “Accuracy” - closeness of the agreement between the result of a measurement and the true value of the quantity being measured

- “Error” - result of a measurement minus the true value of the quantity being measured
“Accuracy” - some related terms

- "Repeatability" - closeness of agreement between the results of successive measurements of the same quantity under the same conditions.

- "Uncertainty" (of measurement) - "parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the quantity being measured."

  i.e. an interval that is likely to contain the 'true value' of the quantity being measured.
Some advantages and disadvantages

Disadvantages

• For “accuracy” and “error” we generally need to know the true value of the quantity being measured

• In general, we don’t know the true value

(If we did, why would we be measuring it ?!)
Disadvantages

“Repeatability” just shows us if we can get the same (or similar) result each time we measure

• It doesn’t tell us if it’s the right value!

• For example, it doesn’t show up systematic errors (biases) in the measurement - just the random ones
More advantages and disadvantages

**Advantages**

“Uncertainty” (of measurement) shows how good a measurement is
- without needing to know the true value
- and includes effects caused by both random and systematic errors

So, from now on, we’ll use *uncertainty* to show how “accurate” our measurements are...
Why is uncertainty so important?

- It gives a quantitative indication of how good a measurement is.
- The measurement result can be used by others (i.e. it is transferable).
- It lets us compare measurement results in a meaningful way.
Comparing measurement results

- Do these results agree?
- How about these?
How to determine uncertainty

Follow these **five** steps:

- Understand the measurement process and identify sources of error
- Quantify each uncertainty contribution
- Evaluate the effect of the uncertainty contribution on the final result
- Combine the contributions to get an overall uncertainty
- Report the result
Example

One-port reflection measurement (magnitude of linear voltage reflection coefficient, VRC) of a coaxial device measured using a Vector Network Analyzer (VNA) at relatively low frequencies (i.e. below 10 GHz)

Assume measured VRC is 0.200
Step 1

Understand the measurement process and identify sources of error

For example, use:

- Measurement model
- Error model
- Computer simulations
- Flow diagrams
An error model was used to represent the VNA:

\[ U = D + T \Gamma + M \Gamma^2 + R_{VRC} \]

This revealed the following sources of error:

- \( D \) = residual directivity
- \( T \) = residual tracking
- \( M \) = residual test port match
- \( R_{VRC} \) = random errors (connector repeatability, noise, etc)

Note: \[ U \] = uncertainty
\[ \Gamma \] = measured VRC (= 0.2)
Step 2

Quantify each uncertainty contribution

- Use standard uncertainties (i.e. ‘standard deviations’)

- Down-convert, if necessary
  for a ‘limit’ (rectangular distribution), divide by \( \sqrt{3} \)
  for a 95% confidence interval, divide by 2
  for a U-shaped distribution, divide by \( \sqrt{2} \)
Example – VNA measurement

• Residual directivity, $D$, and test port match, $M$, evaluated using the 'ripple' technique

\[ D = 0.010 \quad M = 0.010 \]

• Assume U-shape distribution
• Standard uncertainties given by:
  \[ u(D) = 0.010 \div \sqrt{2} = 0.0071 \]
  \[ u(M) = 0.010 \div \sqrt{2} = 0.0071 \]
Example – VNA measurement

- Residual tracking, $T$, based on the manufacturer’s value (i.e. a specification ‘limit’)
  \[ T = 0.001 \]

- Assume a rectangular distribution
- Standard uncertainty given by:
  \[ u(T) = \frac{0.001}{\sqrt{3}} = 0.0006 \]
Evaluate the effect of the uncertainty contribution on the final result

For example, use:
• Partial differentiation (of a measurement model)
• Error models
• Practical experience & experiments
• Monte Carlo methods (computerised statistical sampling)
Example – VNA measurement

The error model:

\[ U = D + T \Gamma + M \Gamma^2 + R_{VRC} \]

gives:

Uncertainty in VRC due to \( u(D) = D = 0.0071 \)
Uncertainty in VRC due to \( u(T) = T \Gamma = 0.0006 \times 0.2 = 0.0001 \)
Uncertainty in VRC due to \( u(M) = M \Gamma^2 = 0.0071 \times 0.2^2 = 0.0003 \)
Combining steps 2 and 3

Monitor variation in a contribution at the same time as monitoring the effects the variation has on the measurement results.

For example:
• Connector repeatability
• Noise
• etc
Example – VNA measurement

- Connector repeatability, noise, etc, evaluated by observing variation in repeat measurements (i.e. repeatability)
  \[ R_{VRC} = 0.010 \]

- Assume a 95% confidence interval
- Standard uncertainty given by:
  \[ 0.010 \div 2 = 0.005 \]

- Uncertainty in VRC due to \(u(R_{VRC}) = R_{VRC} = 0.005\)
Step 4

- Combine the contributions to give the overall uncertainty (called “expanded uncertainty”)
- Use “Root-Sum-of-Squares” (RSS) if contributions are independent (as is usually the case)
- Use linear addition for contributions that may be correlated (i.e. inter-related)
- Expand the uncertainty interval (i.e. multiple by a “coverage factor”)

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Example – VNA measurement

- Residual directivity, $D$, and test port match, $M$, may be correlated, so:
  
  $$u(D) + u(M) = 0.0071 + 0.0003 = 0.0074$$

- Then combine using RSS:
  
  $$\sqrt{(u(D) + u(M))^2 + u(T)^2 + u(R_{VRC})^2} = \sqrt{0.0074^2 + 0.0001^2 + 0.005^2} = 0.0090$$
Example – VNA measurement

- Combined standard uncertainty = 0.009

- Now, expand the standard uncertainty interval to provide a larger interval at a specified level of confidence

- E.g. usually, to achieve a 95% level of confidence, use a coverage factor of two to expand the uncertainty interval

- I.e. Expanded uncertainty, \( U = 0.009 \times 2 = 0.018 \)

- This is the uncertainty to quote to most users of the measurement result (customers, colleagues, etc)
Step 5

Reporting the result
• Unambiguous
• Easy to understand and use

Give:
• Measurement result
• Expanded uncertainty alongside result
• Coverage factor value
• Level of confidence
Example – VNA measurement

Report the result as:

**Linear VRC = (0.200 ± 0.018)**

“The reported uncertainty is an expanded uncertainty using a coverage factor of $k = 2$, providing a level of confidence of approximately 95%.”
Comparing coax and waveguide measurements

Uncertainty components due to terms in the error model relate directly to quality of cal standards and cal method.

These (systematic) uncertainty components will often be smaller in waveguide compared with coax.

Uncertainty components due to random errors relate directly to quality of connections and cable flexure.

These (random) uncertainty components will often be smaller in waveguide compared with coax (assuming the same amount of cable flexure).

For measurements made at similar frequencies, overall uncertainties will often be smaller in waveguide compared with coax.
Conclusions

- Evaluating uncertainty can be easy
- Evaluating uncertainty can be difficult
- Evaluating uncertainty usually involves estimates and guesses based on experience
- Uncertainty estimates can/should be re-evaluated in the light of new work and/or experience
- Keep it a 'living' process

*Often called “the GUM”. This gives a detailed and comprehensive account of general principles involved in evaluating uncertainty of measurement.*


*Gives general information, based on the GUM, for evaluating and expressing uncertainty.*


*Gives guidelines for evaluating the uncertainty of VNA measurements, and includes specific examples. The example given in this lecture is based on Example 1 from this document.*