Microwave Power Measurement and Uncertainty

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Overview

• Microwave power measurement
• Power sensor calibration
• Accuracy of power sensor calibration
• Traceability and primary power standards
• Uncertainty evaluations
• NMC results
Measurement of Microwave Power

- Many instruments can measure RF & μW Power
- Most accurate one is the power meter and sensor combination
- The simplest way is the direct measurement

![Diagram showing power measurement setup]

Measurement of Microwave Power

- Power sensor converts the incident microwave power into a DC or low frequency voltage.
- Three types of power conversion devices

![Diagram showing power sensor and meter setup]
**Power sensor types and operating power range**

![Power sensor types and operating power range diagram](image)

**Accuracy of power measurement**

- Microwave power measurement accuracy depends on the **accuracy** of power sensor.
- The accuracy of the power sensor relies on the **calibration** of the power sensor.
- Microwave power sensor calibration determines the power measurement accuracy and is essential for instrument calibration and system design.
Calibrate power sensor by Comparison

\[ \frac{P_L}{P_M} = c \frac{1 - |\Gamma_L|^2}{1 - |\Gamma_M|^2} \frac{1 - |\Gamma_G|^2}{1 - |\Gamma_D|^2} \]

- incident power to sensor:

\[ P_{inc} = \frac{1}{|1 - \Gamma_G \Gamma_L|^2} P_{gZ_0} \]

\( P_{gZ_0} \) is the power delivered by the generator to a \( Z_0 \) load

Direct comparison transfer

- Load could be another power sensor and meter being calibrated
**Direct comparison transfer for calibration of power sensors**

- Connect each in turn
- If $\Gamma_{std} = \Gamma_{DUT}$, exactly the same amount of power is absorbed for both std and DUT
- Transfer the parameters from std to DUT, complete the calibration

![Diagram showing signal generator, Reference power sensor with meter, DUT power sensor with meter, and equations for $\Gamma_{G}$, $\Gamma_{std}$, $\Gamma_{DUT}$]

**Calculation example**

- Then incident power to standard sensor and to DUT sensor are:

  $P_{inc,Std} = \frac{1}{|1-\Gamma_G \Gamma_{std}|^2} P_{gzo}$
  $P_{inc,DUT} = \frac{1}{|1-\Gamma_G \Gamma_{DUT}|^2} P_{gzo}$

- Substitute for $P_{gzo}$ in $P_{inc,DUT}$ using $P_{inc,Std}$ equation

  $P_{inc,DUT} = P_{inc,Std} \times \frac{|1-\Gamma_{std} \Gamma_G|^2}{|1-\Gamma_{DUT} \Gamma_G|^2}$
  Let $MM1 = \frac{|1-\Gamma_{std} \Gamma_G|^2}{|1-\Gamma_{DUT} \Gamma_G|^2}$

- So, $P_{inc,DUT} = P_{inc,Std} \times MM1$

Say $\Gamma_{std} = 0.1 \exp(j\pi/4)$ [SWR=1.22], $\Gamma_G = 0.25 \exp(-j\pi/2)$ [SWR=1.67], $\Gamma_{DUT} = 0.2 \exp(j\pi)$ [SWR=1.5]

$MM1 = \frac{0.9653}{1.0025} = 0.9629$

If $P_{inc,Std} = 9.5 \text{ mW}$, $P_{inc,DUT} = 9.5 \times 0.9629 = 9.15 \text{ mW}$
Accurate Direct comparison transfer of sensor calibration

\[ F_{EG} = S_{22} - S_{12} \frac{S_{23}}{S_{13}} \]

Effective Source output

Summary of direct comparison method

The best comparison method

- Since add power splitter as an intermediate component, the characterization of signal source is eliminated
- Since use the same arm of the power splitter, the imbalance of the two arms is avoided
- Since magnitude and phase information are used in the comparison, the mismatch uncertainty reduced to minimum
- Since power is monitored, the power drift during comparison can be corrected
**Power Measurements Hierarchy**

National Standards and Traceability (establish the unbroken chain)

- Microcalorimeter, calorimeter and national reference standards
  - NMC Singapore and other NMIs

- Transfer standards
  - NMC Singapore and other NMIs
  - Commercial Standard Laboratories

- Measurement reference standards
  - Manufacturing facilities

- Working standards

- Test equipment
  - Users

**National Standards and Traceability**

- 10 mW, N type connector reference standard from 50MHz to 18GHz calibrated by microcalorimeter of NMC with an uncertainty range from 0.25% to 0.45%

- 10mW, 3.5mm connector reference standards from 10MH to 26.5GHz calibrated by calorimeter of NMC with an uncertainty range from 0.5% to 1.8%

- 10mW, N type connector working standards from 100kHz to 4.2GHz traced to NIST/NPL/PTB power standards with an uncertainty range from 0.25% to 0.5%

- 1mW, 2.4mm connector working standards from 50 MHz to 50GHz traced to NPL/NIST power standards with an uncertainty range from 0.3% to 5%
**Primary standard – calorimeter and microcalorimeter**

- Calorimeters are heat-measuring instruments
- They form the basis of the vast majority of primary standards for RF and microwave power
- In national metrology institutes, mainly 2 types of calorimeters
  - Calorimeter
  - Microcalorimeter
- There are also other types of calorimeters, such as flow calorimeter, adiabatic calorimeter, etc. which are either investigated in early microwave years, or not used popularly.
- Recent researches on quantum-based Rabi oscillation magnetic field measurement by flipping of Cs atoms between two states

**Principle of calorimeter**

![Principle of the dry load calorimeter](image)

**Essential components for calorimeter**

- **Load** in which the power is dissipated
- a thermally isolating transmission line which connects the input to the load
- a temperature sensor
**Dual load calorimeter**

- one of the loads functions as the absorber while a second load acts as a temperature reference
- temperature sensor is mounted on the outside of the load in a position where it cannot be influenced directly by the electromagnetic fields, which is essential for high accuracy

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**Primary Power Standard Operation principle**

- Operation principle of Micro-calorimeters
  - After applying an equivalence correction, both DC and absorbed microwave power generate the same heat
  - Comprehensive and exhaustive analysis is required to determine the equivalence correction and account for all possible thermal and RF errors, such as losses in the transmission lines and the effect of different thermal paths within the micro-calorimeter and the reference standard
  - The DC-substitution technique is used because the fundamental power measurement can then be based on DC voltage (or current) and resistance standards
  - The traceability path leads through the microcalorimeter (for effective efficiency, a unit-less correction factor) and finally back to the national DC standards
Calibration factor, reflection coefficient and linearity

- All power sensors share three important performance factors:

  Reflection coefficient – measured by a network analyzer, preferably a vector network analyzer. If known, it can be used to correct for mismatch errors.

  Linearity – assumed by design and by physics. Can be measured as well.

  Calibration factor – measured with respect to a reference sensor, which is calibrated by a standards laboratory or National Metrology Institutes (NMIs)

Effective Efficiency

- A special term, effective efficiency $\eta_e$, has been adopted for power sensors. Effective efficiency is defined by:

  $\eta_e = \frac{P_{\text{sub}}}{P_{\text{gl}}}$

  Or generally

  $\eta = \frac{P_{\text{DC}}}{P_{\text{RF}}}$

- $P_{\text{gl}}$ is the net RF power absorbed by the sensor during measurement (not incident power)

- $P_{\text{sub}}$ is the DC or low frequency equivalent power, generating the same heat effect or same voltage output as the RF power being measured
Transfer Effective Efficiency

The transfer equation is derived from the definition of effective efficiency

\[
\eta_{DUT} = \frac{P_{DC,DUT}}{P_{RF,DUT}} = \frac{P_{DC,DUT}}{P_{DC,Std}} \times \frac{P_{RF,Std}}{P_{RF,Std} - \left(1 - |\Gamma_G^{DUT}|^2\right)\left(1 - |\Gamma_G^{Std}|^2\right)}
\]

\[
= \frac{P_{DC,DUT}}{P_{DC,Std}} \times \frac{1 - |\Gamma_G^{Std}|^2}{1 - |\Gamma_G^{DUT}|^2} \times \frac{1 - |\Gamma_G^{DUT}|^2}{1 - |\Gamma_G^{Std}|^2}
\]

\[P_{RATIO} \times \text{MisMatch factor}\]

Transfer Effective Efficiency

Effective efficiency \(\eta_{Std}\) from a standard thermistor to a DUT sensor is transferred using equation

\[\eta_{DUT} = \eta_{Std} \times P_{RATIO} \times MM\]

\(\eta_{DUT}\) is the effective efficiency of DUT sensor

\(\eta_{Std}\) is the effective efficiency of a standard sensor, it comes from a national lab, a calibration lab, or a manufacturer

\(P_{RATIO}\) is a power ratio, depending on the system setup, its value shows later

\(MM\) is the mismatch factor
Transfer Effective Efficiency

- The mismatch factor at this time is

\[ MM = \frac{1 - |\Gamma_{Std}|^2}{1 - |\Gamma_{DUT}|^2} \frac{|1 - \Gamma_{DUT} \Gamma_G|^2}{|1 - \Gamma_{Std} \Gamma_G|^2} \]

\( \Gamma_{DUT}, \Gamma_{Std}, \) and \( \Gamma_G \) are the respective complex reflection coefficients of the DUT, the standard, and the generator.
\( \Gamma_{DUT} \) and \( \Gamma_{Std} \) can be measured by vector network analyzer.

Calibration factor

- The calibration factor

\[ K_{DUT} = \eta_{DUT} (1 - |\Gamma_{DUT}|^2) \]

Which combines effective efficiency and mismatch loss of the DUT

- The calibration factor of DUT can be obtained if known \( \eta_{DUT} \) and \( \Gamma_{DUT} \).
Calibration factor transfer

- In normal calibration lab practice, the known standard sensor’s parameter is calibration factor, rather than effective efficiency.
- Calibration factor $K_{Std}$ from standard sensor to a DUT sensor is transferred using

$$
K_{DUT} = K_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{1 - \Gamma_{DUT} \Gamma_{EG}}{1 - \Gamma_{Std} \Gamma_{EG}}
$$

$$
= K_{Std} \times P_{RATIO} \times MM1
$$

$K_{DUT}$ is the calibration factor of DUT sensor

$K_{Std}$ is the calibration factor of a standard sensor, it comes from a national lab, a calibration lab, or a manufacturer

If the system set up is the same as effective efficiency transfer, $P_{RATIO}$ is the same

$MM1$ is the mismatch factor, it is different from MM

Uncertainty evaluation

- According to GUM, the uncertainties of $\eta_{DUT}$ and $K_{DUT}$ in previous equations are propagated from the uncertainties of the standards, power meter readings and mismatch items which accumulated the uncertainties of various reflection coefficients. The combined uncertainty $u_c$ can be derived by

$$
 u_c^2 (y_{DUT}) = \sum_{n=1}^{M} c_n^2 u_n^2
$$

where $c_n$ are the sensitivity coefficients and $u_n$ are the individual uncertainty contributions.

- The sensitivity coefficients $c_n$ are the partial derivative with respect to the variables
Uncertainty evaluation

- As reflection coefficient is a complex term and mismatch factors are the algebra sum of the reflection coefficients, the microwave power measurement uncertainty calculation becomes a complicated issue.
- also discussions in the expression either in polar or rectangular coordinates
- With \( \Gamma \) expressed in terms of magnitudes/phases and expressed in terms of real/imaginary, we derived the sensitivity coefficients in the measurement uncertainty calculation for calibration of microwave power sensor in three cases,
  - to obtain the effective efficiency \( \eta_{DUT} \) from a known efficiency \( \eta_{Std} \);
  - to obtain the calibration factor \( K_{DUT} \) from a known calibration factor \( K_{Std} \);
  - the \( K_{DUT} \) from a known \( \eta_{Std} \).

Model expressed in magnitude and phase

\[
\eta_{DUT} = \eta_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{Std}}{P_{DUT}} \times \frac{1 - |\Gamma_{Std}|^2}{1 - |\Gamma_{DUT}|^2} \times \frac{1 - |\Gamma_{DUT} \Gamma_{EG}|^2}{1 - |\Gamma_{Std} \Gamma_{EG}|^2}
\]

\[
= \eta_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{Std}}{P_{DUT}} \times \frac{1 - |\Gamma_{Std}|^2}{1 - |\Gamma_{DUT}|^2} \times \frac{1 + |\Gamma_{DUT} \Gamma_{EG}|^2 - 2 |\Gamma_{DUT} \Gamma_{EG}| \cos(\theta_{DUT} + \theta_{EG})}{1 + |\Gamma_{Std} \Gamma_{EG}|^2 - 2 |\Gamma_{Std} \Gamma_{EG}| \cos(\theta_{Std} + \theta_{EG})}
\]

\[
K_{DUT} = K_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{Std}}{P_{DUT}} \times \frac{1 - |\Gamma_{DUT} \Gamma_{EG}|^2}{1 - |\Gamma_{Std} \Gamma_{EG}|^2}
\]

\[
= K_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{Std}}{P_{DUT}} \times \frac{1 + |\Gamma_{DUT} \Gamma_{EG}|^2 - 2 |\Gamma_{DUT} \Gamma_{EG}| \cos(\theta_{DUT} + \theta_{EG})}{1 + |\Gamma_{Std} \Gamma_{EG}|^2 - 2 |\Gamma_{Std} \Gamma_{EG}| \cos(\theta_{Std} + \theta_{EG})}
\]
Uncertainty evaluation

\[
\begin{align*}
kkM_{1_{in}} &= \frac{\partial M_1}{\partial \Gamma_{Std}} \cdot \frac{M_1}{M_{1p}} \times 2A(AC - E) \\
kkM_{1_{ac}} &= \frac{\partial M_1}{\partial \theta_{ac}} = \frac{M_1}{M_{1p}} \times 2AC \sin(\theta_{ac} + \theta_{EC}) \\
kkM_{1_{avr}} &= \frac{\partial M_1}{\partial \Gamma_{DUT}} = \frac{1}{M_{1p}} \times 2A(AB - D) \\
kkM_{1_{pwv}} &= \frac{\partial M_1}{\partial \theta_{EG}} = \frac{2AB}{M_{1p}} \sin(\theta_{EC} + \theta_{EG}) \\
kkM_{1_{ine}} &= \frac{\partial M_1}{\partial \Gamma_{EG}} = \frac{2B}{M_{1p}} \times (AB - D) - \frac{M_{1p}}{(M_{1p})^2} \times (AC - E) \\
kkM_{1_{pwe}} &= \frac{\partial M_1}{\partial \theta_{EG}} = \frac{2AB}{M_{1p}} \sin(\theta_{EC} + \theta_{EG}) - \frac{M_{1p}}{(M_{1p})^2} \times 2AC \sin(\theta_{ac} + \theta_{EG}) \\
&= kkM_{1_{pwv}} + kkM_{1_{pwe}}
\end{align*}
\]

Uncertainty evaluation

Model expressed in real and imaginary

\[
\begin{align*}
A &= \Re\{\Gamma_{Std}\} = \Gamma_{Std-\text{Re}}, \quad B = \Im\{\Gamma_{Std}\} = \Gamma_{Std-\text{Im}}. \\
C &= \Re\{\Gamma_{DUT}\} = \Gamma_{DUT-\text{Re}}, \\
D &= \Im\{\Gamma_{DUT}\} = \Gamma_{DUT-\text{Im}}, \\
E &= \Re\{\Gamma_{EG}\} = \Gamma_{EG-\text{Re}} \quad \text{and} \quad F = \Im\{\Gamma_{EG}\} = \Gamma_{EG-\text{Im}}.
\end{align*}
\]

\[
\begin{align*}
\eta_{DUT} &= \eta_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{3Std}}{P_{3DUT}} \times \frac{1 - |\Gamma_{Std}|^2}{1 - |\Gamma_{DUT}|^2} \times \frac{1 - |\Gamma_{DUT\Gamma_{EG}}|^2}{1 - |\Gamma_{Std\Gamma_{EG}}|^2} \\
&= \eta_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{3Std}}{P_{3DUT}} \times \frac{1 - A^2 - B^2}{1 - C^2 - D^2} \times \frac{1 + 2DF - 2CE + C^2E^2 + D^2E^2 + C^2F^2 + D^2F}{1 + 2BF - 2AE + A^2E^2 + B^2E^2 + A^2F^2 + B^2F^2} \\
K_{DUT} &= K_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{3Std}}{P_{3DUT}} \times \frac{|1 - \Gamma_{DUT\Gamma_{EG}}|^2}{|1 - \Gamma_{Std\Gamma_{EG}}|^2} \\
&= K_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{3Std}}{P_{3DUT}} \times \frac{1 + 2DF - 2CE + C^2E^2 + D^2E^2 + C^2F^2 + D^2F}{1 + 2BF - 2AE + A^2E^2 + B^2E^2 + A^2F^2 + B^2F^2}
\end{align*}
\]
Measurement result

- At NMC, results obtained for calibration factor and the measurement uncertainties of a power sensor (8481A) calibrated by a power standard (CN36) in the frequency range 50 MHz to 18 GHz using the direct comparison transfer method with Weinshel 1870A splitter. The effective efficiency of power standard was measured using a micro-calorimeter. The expanded uncertainties are less than 0.9% for the frequency range at $k=2$.

Uncertainty evaluation

- Limitations to the GUM uncertainty framework
- To tackle the problem, numerical methods using Monte Carlo Method (MCM) was proposed and described in the document “Evaluation of measurement data – Supplement 1 to the GUM – Propagation of distributions using a Monte Carlo method” (Supplement 1)
- Evaluation of measurement data – Supplement 2 to the GUM – Models with any number of output quantities (Supplement 2) treats multivariate measurement models (2009-06-11 draft)
References


[5] Ronald Ginley, A direct comparison system for measuring radio frequency power (100kHz to 18GHz), MEASURE, Vol.1, No.4, December 2006

